Generic Partition Refinement and Weighted Tree Automata

Thorsten Wißmann `thorsten-wissmann.de`

FM'19 Best Theory Paper, coauthored by:
Hans-Peter Deifel, Stefan Milius, Lutz Schröder

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ICSE Showcase 2023
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Generic Partition Refinement
and

and

Weighted Tree Automata

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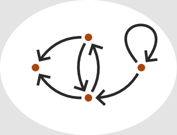
Weighted Tree Automata

Generic Partition Refinement

and

Algorithm for
state equivalence:



Weighted Tree Automata

Generic Partition Refinement

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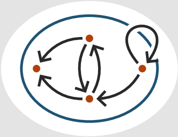

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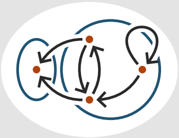

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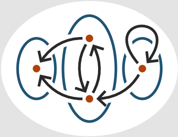

Weighted Tree Automata

Generic Partition Refinement

and

- Deterministic Automata
- LTS/Bisimilarity
- Markov Chains

⋮

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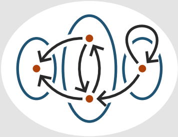

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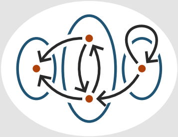

and

Weighted Tree Automata

Application: Equivalence Checking

Application in Model Checking

Similar Ideas

Similar Run-Time

System type hard-wired

Similar Ideas

Similar Run-Time

Application: Equivalence Checking

Deterministic Finite Automata

Hopcroft '71 Gries '73 Knuutila '01

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(Labelled) Transition Systems

Paige, Tarjan '87 Valmari '09

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"Markov Chain Lumping"

Valmari, Franceschinis '10

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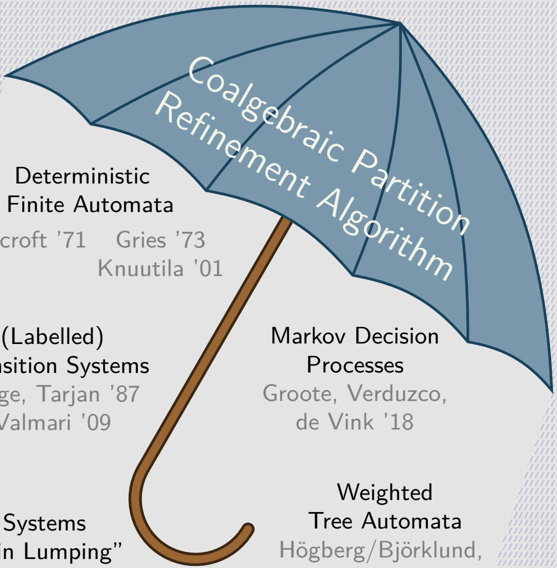
Weighted Tree Automata
Högberg/Björklund, Maletti, May '07



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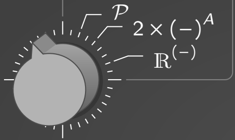
a category

state space

Functor $F\colon \mathcal{C} \to \mathcal{C}$

$F\colon \mathsf{Set} \to \mathsf{Set}$



F-Coalgebra

$$c\colon C \to FC$$

Example

F-Coalgebra-Homomorphism

$$h\colon (C, c) \to (D, d)$$

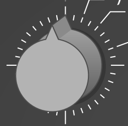
Induced Equivalence Notion:

a category

state space

Functor $F\colon \mathcal{C} \to \mathcal{C}$

$F\colon \mathsf{Set} \to \mathsf{Set}$



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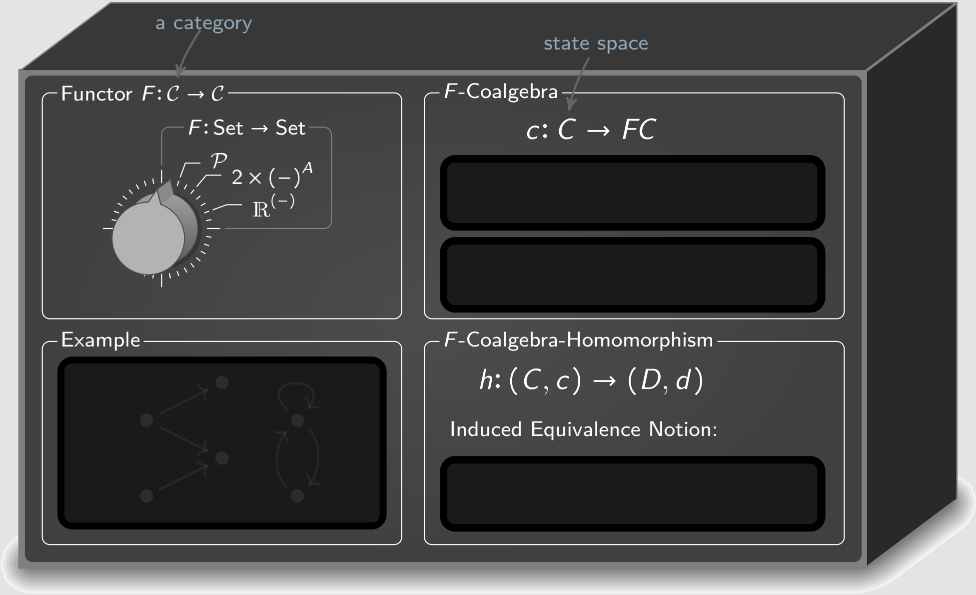
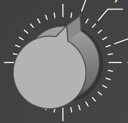

a category
state space
Functor $F\colon \mathcal{C} \to \mathcal{C}$
$F\colon \mathsf{Set} \to \mathsf{Set}$
$\mathcal{P}$
$2 \times (-)^A$
$\mathbb{R}^{(-)}$
F-Coalgebra
$c\colon C \to FC$
Example
F-Coalgebra-Homomorphism
$h\colon (C,c) \to (D,d)$
Induced Equivalence Notion:

a category

Functor $F\colon \mathcal{C} \to \mathcal{C}$

$F\colon \mathsf{Set} \to \mathsf{Set}$



- $\mathcal{P}$
- $2 \times (-)^A$
- $\mathbb{R}^{(-)}$

state space

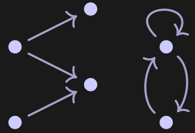
F-Coalgebra

$$c\colon C \to FC$$

$$c\colon C \to \mathcal{P}C$$

"Transition System"

Example



F-Coalgebra-Homomorphism

$$h\colon (C, c) \to (D, d)$$

Induced Equivalence Notion:

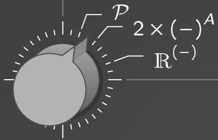
"Strong Bisimilarity"

a category

state space

Functor $F\colon \mathcal{C} \to \mathcal{C}$

$F\colon \mathsf{Set} \to \mathsf{Set}$



F-Coalgebra

$$c\colon C \to FC$$

Example

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$$h\colon (C, c) \to (D, d)$$

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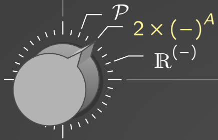


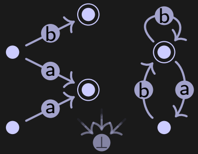
F-Coalgebra

$$c\colon C \to FC$$

$$c\colon C \to 2 \times C^A$$

"Deterministic automaton"

Example



F-Coalgebra-Homomorphism

$$h\colon (C, c) \to (D, d)$$

Induced Equivalence Notion:

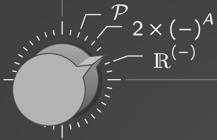
"Same Language"

a category

state space

Functor $F\colon \mathcal{C} \to \mathcal{C}$

$F\colon \mathrm{Set} \to \mathrm{Set}$



F-Coalgebra

$$c\colon C \to FC$$

Example

F-Coalgebra-Homomorphism

$$h\colon (C, c) \to (D, d)$$

Induced Equivalence Notion:

a category

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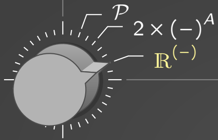


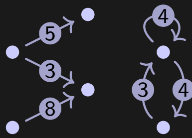
F-Coalgebra

$$c\colon C \to FC$$

$$c\colon C \to \mathbb{R}^{(C)}$$

"Markov Chain"

Example



F-Coalgebra-Homomorphism

$$h\colon (C, c) \to (D, d)$$

Induced Equivalence Notion:

"Weighted Bisimilarity"

Instances of
F-Coalgebras

Transition Systems

Deterministic Automata

Markov Chains

Coalgebraic
Results

Coinduction Principle

Logics

Constructions

Instances of F-Coalgebras

Coalgebraic Results

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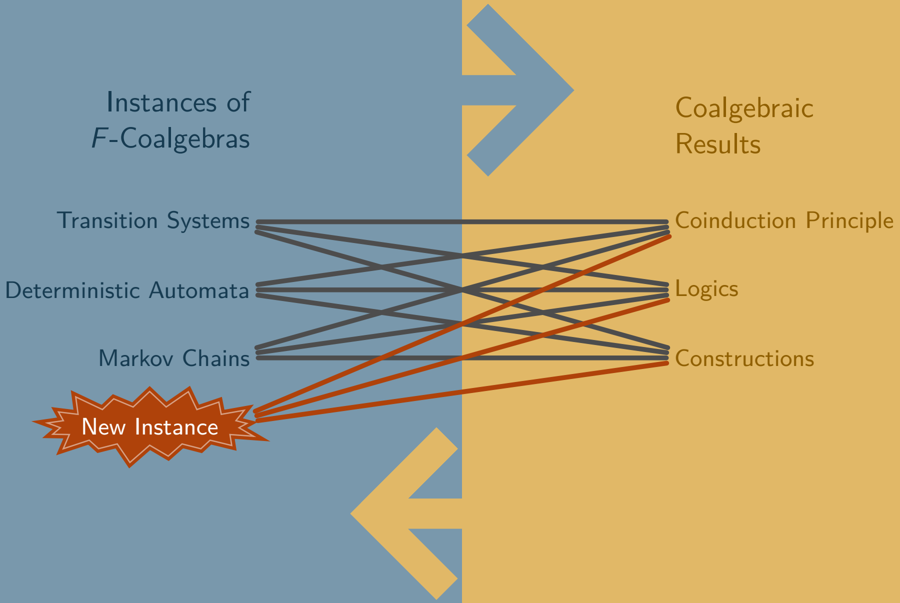
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New Instance
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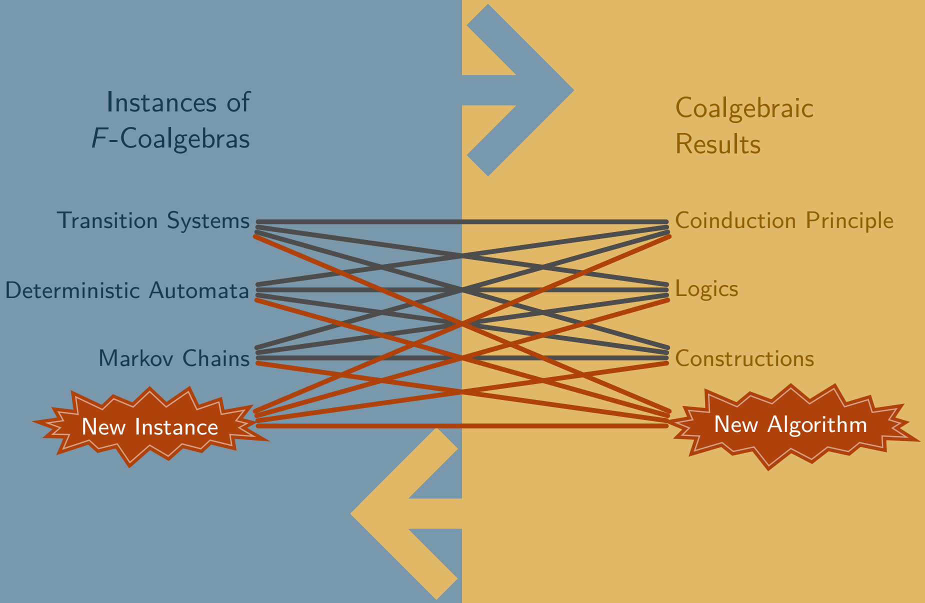
Instances of
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Functor F
e.g.
Automaton
F-coalgebra
Coalgebraic
Partition
Refinement
Behaviourally
Equivalent
States
e.g.
minimized
Automaton

New Algorithm

Theorem: Coalgebraic Partition Refinement

If F is "zippable" & is implements our interface, our algorithm correctly minimizes an F-coalgebra in time:

Transitions (m), States (n)

$$\mathcal{O}(m \cdot \log n) \qquad (m \geq n)$$

New Algorithm

Theorem: Coalgebraic Partition Refinement

If F is "zippable" & is implements our interface, our algorithm correctly minimizes an F-coalgebra in time:

Transitions → m, States → n

$$\mathcal{O}(m \cdot \log n) \qquad (m \geq n)$$

Implementation

- CoPaR
- in Haskell
- can handle large input files (200MB)

System Type	Functor FX	Run time $(m \geq n)$		Dedicated Algorithm	
Transition Systems	$\mathcal{P}_\mathsf{f}X$	$m \cdot \log n$	=	$m \cdot \log n$	Paige, Tarjan 1987
Markov Chains	$\mathbb{R}^{(X)}$	$m \cdot \log n$	=	$m \cdot \log n$	Valmari, Franceschinis 2010
Deterministic Automata	$2 \times X^A$ (A fixed)	$n \cdot \log n$	=	$n \cdot \log n$	Hopcroft 1971
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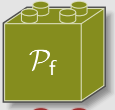
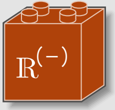
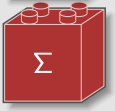


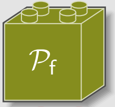
Powerset Functor
Non-Determinism & Bisimilarity



Distribution Functor
Probabilistic Behaviour



Signature Functor
Automata & Infinite Trees



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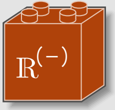

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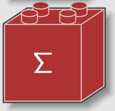

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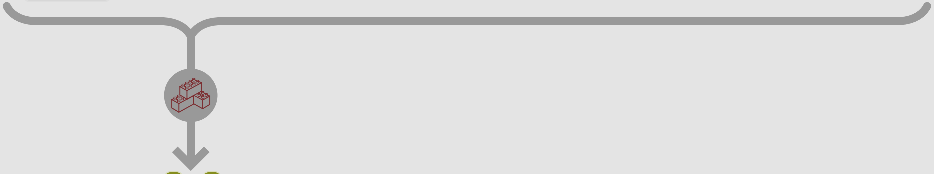
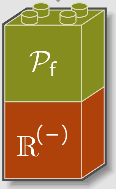
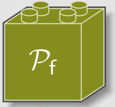




Coalgebras = Markov Decision Processes / Segala Systems



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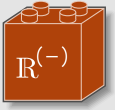

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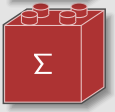

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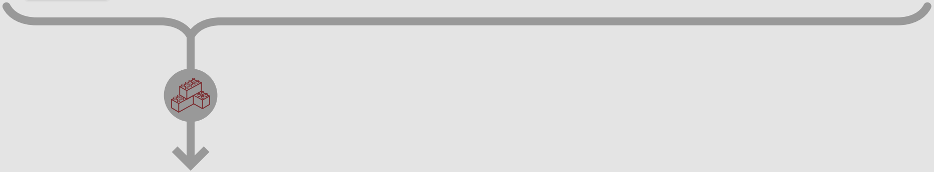

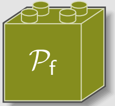

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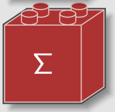
Non-Determinism & Bisimilarity



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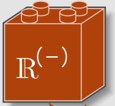

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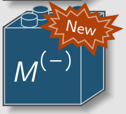
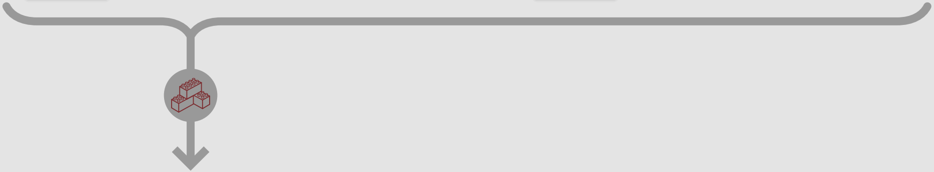
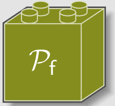
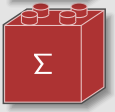


Monoid-valued Functor
Weighted Automata for General Monoids





Powerset Functor
Non-Determinism & Bisimilarity



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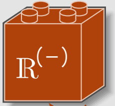
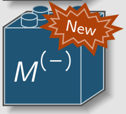
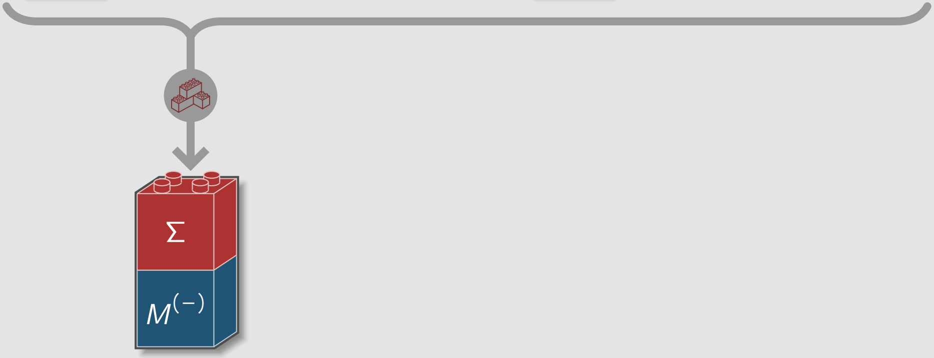
Automata & Infinite Trees



Distribution Functor
Probabilistic Behaviour



Monoid-valued Functor
Weighted Automata for General Monoids



$\mathcal{P}_f$

Powerset Functor
Non-Determinism & Bisimilarity

$\mathbb{R}^{(-)}$

Distribution Functor
Probabilistic Behaviour

Σ

Signature Functor
Automata & Infinite Trees

New

$M^{(-)}$

Monoid-valued Functor
Weighted Automata for General Monoids

Σ

$M^{(-)}$

Theorem

New Instance

Coalgebras for $M^{\Sigma(-)}$

$=$

Weighted Tree Automata with backwards bisimilarity (as considered by Högberg/Björklund, Maletti, May '07)

System Type	Functor FX	Run time $(m \geq n)$		Dedicated Algorithm	
Transition Systems	$\mathcal{P}_{\mathsf{f}}X$	$m \cdot \log n$	=	$m \cdot \log n$	Paige, Tarjan 1987
Markov Chains	$\mathbb{R}^{(X)}$	$m \cdot \log n$	=	$m \cdot \log n$	Valmari, Franceschinis 2010
Deterministic Automata	$2 \times X^A$ $(A \text{ fixed})$	$n \cdot \log n$	=	$n \cdot \log n$	Hopcroft 1971
Colour Refinement	$\mathcal{B}X = \mathbb{N}^{(X)}$	$m \cdot \log n$	=	$m \cdot \log n$	Berkholz, Bonsma, Grohe 2017

System Type	Functor FX	Run time $(m \geq n)$		Dedicated Algorithm	
Transition Systems	$\mathcal{P}_f X$	$m \cdot \log n$	$=$	$m \cdot \log n$	Paige, Tarjan 1987
LTS	$\mathcal{P}_f(\mathbb{N} \times X)$	$m \cdot \log m$	$=$	$m \cdot \log m$	Dovier, Piazza, Policriti 2004
			$>$	$m \cdot \log n$	Valmari 2009
Markov Chains	$\mathbb{R}^{(X)}$	$m \cdot \log n$	$=$	$m \cdot \log n$	Valmari, Franceschinis 2010
Deterministic Automata	$2 \times X^A$ $(A \text{ fixed})$	$n \cdot \log n$	$=$	$n \cdot \log n$	Hopcroft 1971
	$2 \times \mathcal{P}_f(A \times X)$	$\lvert A\rvert \cdot n \cdot \log n$	$=$	$\lvert A\rvert \cdot n \cdot \log n$	Gries 1973/Knuutila 2001
Markov Decision Processes (Segala Systems)	$\mathcal{P}_f(A \times \mathcal{D}X)$	$m_{\mathcal{D}} \cdot \log m_{\mathcal{P}_f}$	$<$	$m \cdot \log n$	Baier, Engelen, Majster-Cederbaum 2000
			$=$	$m_{\mathcal{D}} \cdot \log m_{\mathcal{P}_f}$	Groote, Verduzco, de Vink 2018
Colour Refinement	$\mathcal{B}X = \mathbb{N}^{(X)}$	$m \cdot \log n$	$=$	$m \cdot \log n$	Berkholz, Bonsma, Grohe 2017
Weighted Tree Automata (Backwards Bisimulation)	$M^{(\Sigma X)}$ $(M \text{ non-cancellable})$	$m \cdot \log^2 m$	$<$	$m \cdot n$	Högberg/Björklund, Maletti, May 2007
	$M^{(\Sigma X)}$ $(M \text{ cancellable})$	$m \cdot \log m$	$=$ Σ fixed	$m \cdot \log n$	Högberg/Björklund, Maletti, May 2007

System Type	Functor FX	Run time $(m \geq n)$		Dedicated Algorithm	
Transition Systems	$\mathcal{P}_f X$	$m \cdot \log n$	=	$m \cdot \log n$	Paige, Tarjan 1987
LTS	$\mathcal{P}_f(\mathbb{N} \times X)$	$m \cdot \log m$	=	$m \cdot \log m$	Dovier, Piazza, Policriti 2004
			>	$m \cdot \log n$	Valmari 2009
Markov Chains	$\mathbb{R}^{(X)}$	$m \cdot \log n$	=	$m \cdot \log n$	Valmari, Franceschinis 2010
Deterministic Automata	$2 \times X^A$ (A fixed)	$n \cdot \log n$	=	$n \cdot \log n$	Hopcroft 1971
	$2 \times \mathcal{P}_f(A \times X)$	$\lvert A \rvert \cdot n \cdot \log n$	=	$\lvert A \rvert \cdot n \cdot \log n$	Gries 1973/Knuutila 2001
Markov Decision Processes (Segala Systems)	$\mathcal{P}_f(A \times \mathcal{D}X)$	$m_{\mathcal{D}} \cdot \log m_{\mathcal{P}_f}$	<	$m \cdot \log n$	Baier, Engelen, Majster-Cederbaum 2000
			=	$m_{\mathcal{D}} \cdot \log m_{\mathcal{P}_f}$	Groote, Verduzco, de Vink 2018
Colour Refinement	$\mathcal{B}X = \mathbb{N}^{(X)}$	$m \cdot \log n$	=	$m \cdot \log n$	Berkholz, Bonsma, Grohe 2017
Weighted Tree Automata (Backwards Bisimulation)	$M^{(\Sigma X)}$ (M non-cancellable)	$m \cdot \log^2 m$	<	$m \cdot n$	Högberg/Björklund, Maletti, May 2007
	$M^{(\Sigma X)}$ (M cancellable)	$m \cdot \log m$	= Σ fixed	$m \cdot \log n$	Högberg/Björklund, Maletti, May 2007

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