Explaining Behavioural Inequivalence Generically in Quasilinear Time



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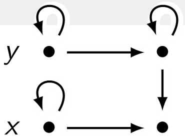
Why are x and y not bisimilar?



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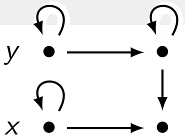

$\Rightarrow$ because $\Box\Diamond\top$ distinguishes them

Generically

In Quasilinear Time

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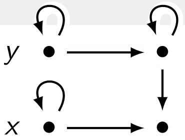

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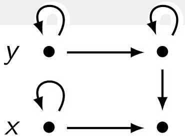

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For different system types

- weighted or probabilistic systems
- (tree) automata

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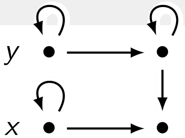

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In Quasilinear Time

$$\mathcal{O}(\overset{\text{transitions}}{m} \cdot \log \overset{\text{states}}{n}) \begin{cases} \text{Formula dag size} \\ \text{Run time} \end{cases}$$

Coalgebra – Generic state based systems

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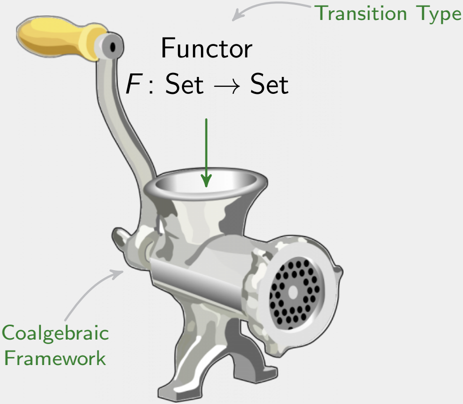
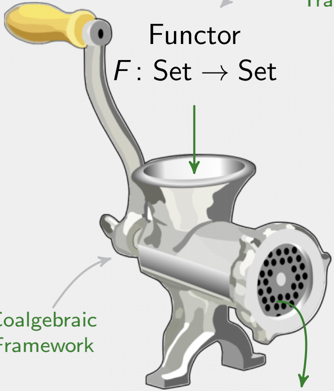


Coalgebra – Generic state based systems

Transition Type

Functor
$F\colon \mathsf{Set} \to \mathsf{Set}$



Coalgebraic Framework

F-Coalgebra
$C \xrightarrow{c} FC$

Coalgebra – Generic state based systems

Powerset $\mathcal{P}$

Transition Type

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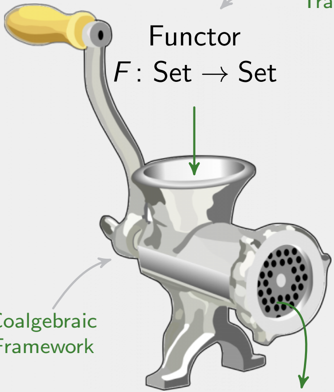

Coalgebraic Framework

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$C \xrightarrow{c} FC$

Transition System

$C \xrightarrow{c} \mathcal{P}C$

Coalgebra – Generic state based systems

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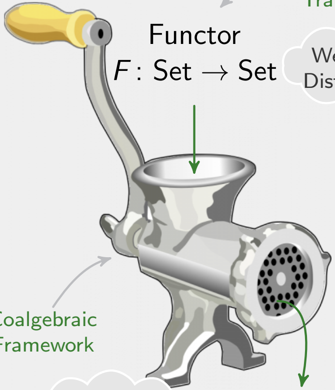
$F\colon \mathsf{Set} \to \mathsf{Set}$

Weights $\mathbb{R}^{(-)}$
Distributions $\mathcal{D}$



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Automata $2 \times (-)^A$

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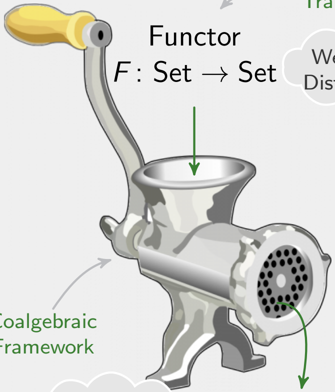

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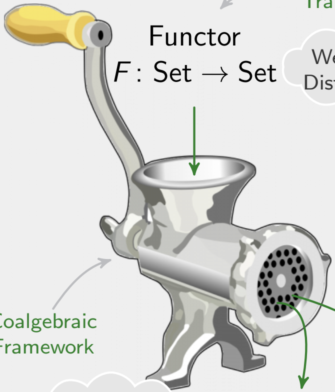

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F-Coalgebra
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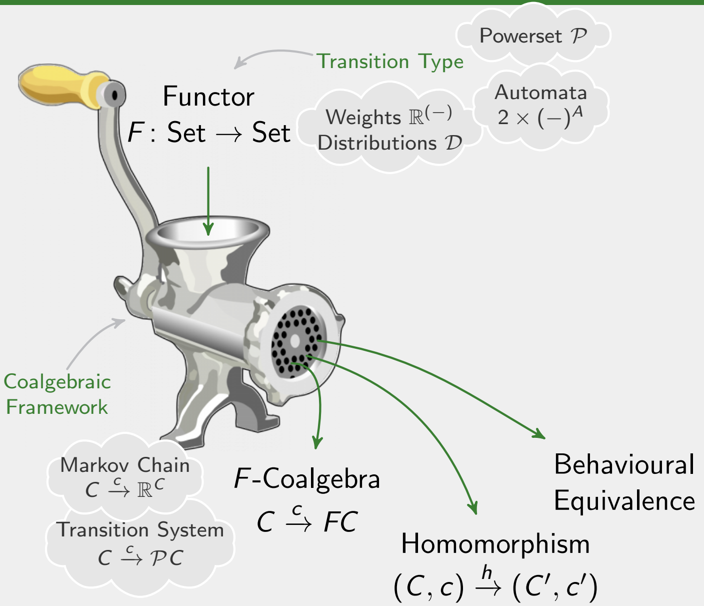
Homomorphism
$(C, c) \xrightarrow{h} (C', c')$

Coalgebra – Generic state based systems



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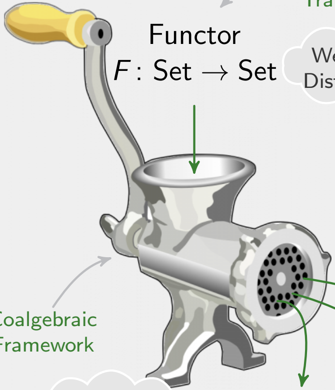

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Behavioural Equivalence

Strong Bisimilarity

Language Equivalence

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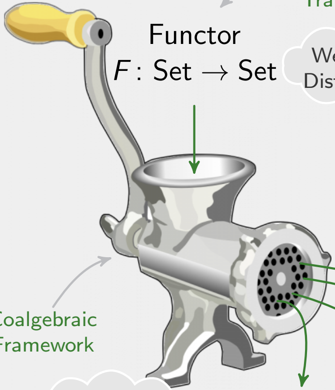

Coalgebraic Framework

Coalgebraic Logic

Behavioural Equivalence

Strong Bisimilarity

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Coalgebraic Logic

Syntax

e.g. $\Lambda = \{\Diamond/1\}$ for $FX = \mathcal{P}X$

For a modal signature Λ, put

$$\phi_1, \ldots, \phi_n ::= \top \mid \phi_1 \wedge \phi_2 \mid \neg\phi \mid \heartsuit(\phi_1, \ldots, \phi_n) \qquad \heartsuit/n \in \Lambda$$

Semantics in $c\colon C \to FC$

$$[\![\top]\!] = C \qquad [\![\phi \wedge \psi]\!] = [\![\phi]\!] \cap [\![\psi]\!] \qquad [\![\neg\phi]\!] = C \setminus [\![\phi]\!]$$

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Definition: Predicate Lifting for $\heartsuit/n \in \Lambda$

Pattinson '03

$=$ Natural transformation $[\![\heartsuit]\!]_X \colon (2^X)^n \to 2^{FX}$

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$$x \in [\![\heartsuit(\phi_1, \dots, \phi_n)]\!] \quad :\Leftrightarrow \quad c(x) \in [\![\heartsuit]\!]_C([\![\phi_1]\!], \dots, [\![\phi_n]\!])$$

Witnesses for (In-)Equivalence

Task
Given x, y in a finite transition system, construct a witness whether x, y are bisimilar

Yes → find bisimulation relating x and y

No → find modal formula distinguishing x and y

Adequacy Theorem for transition systems

x, y bisimilar $\Longrightarrow$ x, y accept the same modal formulae

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Adequacy Theorem for F-coalgebras

x, y beh. equivalent $\Longrightarrow$ x, y accept the same coalgebraic formulae

Witnesses for (In-)Equivalence

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Given x, y in a finite F-coalgebra, construct a witness whether x, y are behavourally equivalent

Yes

find coalgebra morphism relating x and y

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find coalgebraic formula distinguishing x and y

Adequacy Theorem for F-coalgebras

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Witnesses for (In-)Equivalence

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Given x, y in a finite F-coalgebra, construct a witness whether x, y are behavourally equivalent

Yes → find coalgebra morphism relating x and y

coalgebraic partition refinement (Concur'17)

No → find coalgebraic formula distinguishing x and y

the present paper

Adequacy Theorem for F-coalgebras

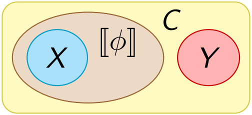
x, y beh. equivalent $\implies$ x, y accept the same coalgebraic formulae

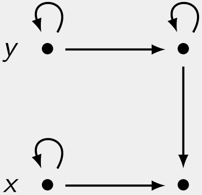
Certificates

For $c\colon C \to FC$, e.g. a transition system with states C

Distinguishing Formula

ϕ distingishes $X \subseteq C$ from $Y \subseteq C$
if $X \subseteq [\![\phi]\!]$ and $[\![\phi]\!] \cap Y = \emptyset$.





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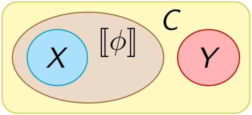

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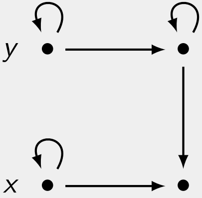

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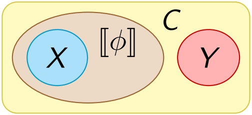

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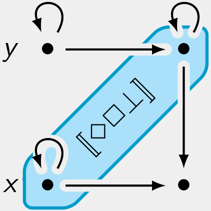

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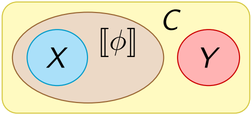

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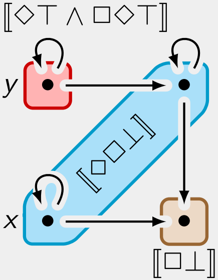


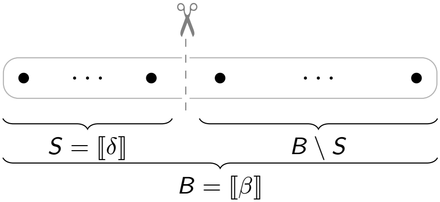
distinguishing formula for every pair $=$ certificates for behavioural equivalence classes

Initially

All states of $c\colon C \to FC$ are grouped w.r.t. $C \xrightarrow{c} FC \xrightarrow{F!} F1$

For example: live states $[\![\Diamond\top]\!]$ and deadlock states $[\![\Box\bot]\!]$.

Refinement Step for $\mathcal{P}$



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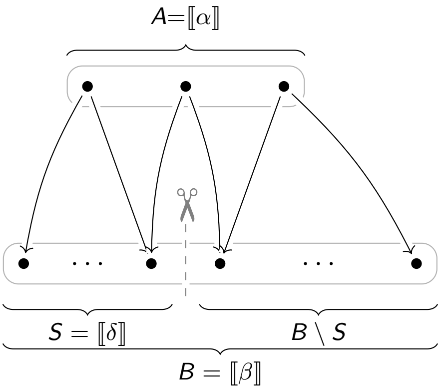

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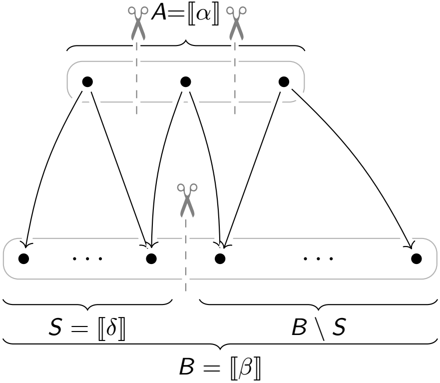

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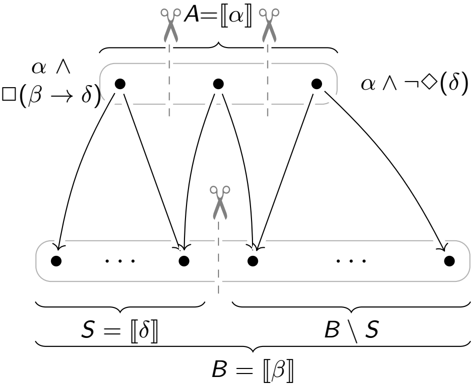

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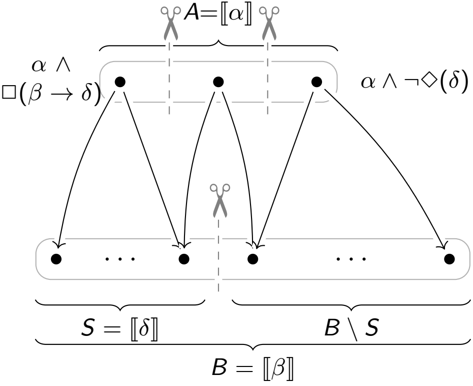

States $x_1, x_2 \in A$ stay together iff

$$\mathcal{P}\chi_S^B(c(x_1)) = \mathcal{P}\chi_S^B(c(x_2)).$$

$$\chi_S^B\colon C \to 3$$

$$\chi_S^B(x) = \begin{cases} 2 & \text{if } x \in S \\ 1 & \text{if } x \in B \setminus S \\ 0 & \text{if } x \notin B \end{cases}$$

$$C \xrightarrow{c} \mathcal{P}C \xrightarrow{\mathcal{P}\chi_S^B} \mathcal{P}3$$

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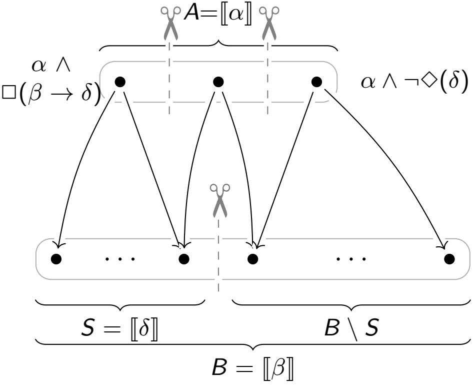
Refinement Step for F



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$$C \xrightarrow{c} FC \xrightarrow{F\chi_S^B} F3$$

Generic Certificates

In the refinement step $S \subsetneq B \subseteq C$, we refine blocks w.r.t.

$$C \xrightarrow{c} FC \xrightarrow{F\chi_S^B} F3$$

Definition: $F3$-modalities

For $F\colon \mathrm{Set} \to \mathrm{Set}$, define the signature

$$\Lambda = \{\ulcorner t \urcorner/2 \mid t \in F3\} \qquad (\ulcorner t \urcorner = \text{syntactic representation})$$

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Semantics

$$\lambda_X^{\ulcorner t \urcorner}\colon (2^X)^2 \to 2^{FX} \qquad \lambda_X^{\ulcorner t \urcorner}(S, B) = \{t' \in FX \mid F\chi_{S\cap B}^B(t') = t\}$$

$$x \in [\![\ulcorner t \urcorner(\delta, \beta)]\!] \iff F\chi_{[\![\delta]\!]}^{[\![\beta]\!]}(c(x)) = t \qquad \text{if } [\![\delta]\!] \subseteq [\![\beta]\!]$$

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Example for $F = \mathcal{P}$

$\ulcorner\{0,2\}\urcorner(\delta,\beta)$ is equivalent to $\overbrace{\Diamond\neg\beta}^{\text{‘0’}} \wedge \neg\overbrace{\Diamond(\beta \wedge \neg\delta)}^{\text{‘1’}} \wedge \overbrace{\Diamond(\delta \wedge \beta)}^{\text{‘2’}}$.

Coalgebraic Partition Refinement with Certificates

Algorithm: during partition refinement for $c\colon C \to FC$

1. Initially: annotate the block of x with the constant:
   $\ulcorner F!(c(x)) \urcorner$ where $t \in F1 \subseteq F3$.

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Algorithm: during partition refinement for $c\colon C \to FC$

1. Initially: annotate the block of x with the constant: $\ulcorner F!(c(x)) \urcorner$ where $t \in F1 \subseteq F3$.
2. Whenever $A = [\![\alpha]\!]$ is split for $S \subsetneq B \subseteq C$, $S = [\![\delta]\!]$, $B = [\![\beta]\!]$: annotate the block of $x \in A$ with: $\alpha \wedge \ulcorner F\chi_S^B(c(x)) \urcorner(\delta, \beta)$

Coalgebraic Partition Refinement with Certificates

Algorithm: during partition refinement for $c\colon C \to FC$

1. Initially: annotate the block of x with the constant:
   $\ulcorner F!(c(x)) \urcorner$ where $t \in F1 \subseteq F3$.
2. Whenever $A = [\![\alpha]\!]$ is split for $S \subsetneq B \subseteq C$, $S = [\![\delta]\!]$, $B = [\![\beta]\!]$:
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Theorem.

If F is zippable [Concur'17] and (C, c) has n states and m transitions, then the algorithm...

1. is correct: every block is annotated with its certificate
2. constructs formulae of dag size: $\mathcal{O}(m \cdot \log n + n)$.
3. requires only constant run-time overhead.

Coalgebraic Partition Refinement

Theorem Wißmann, Dorsch, Milius, Schröder '20

If F is zippable and has a refinement interface, then coalgebraic partition refinement runs in

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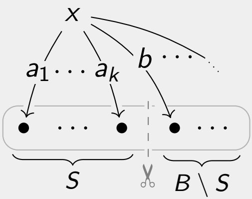
Examples

Automata, Tree Automata, Transition Systems, Weighted Systems, Markov chains, $\cdots$



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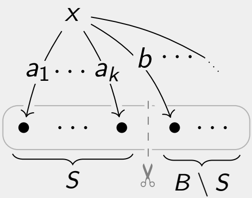

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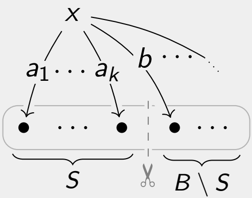
$$\langle F\chi_{\{1,2\}}, F\chi_{\{2\}}\rangle\colon F3 \to F2 \times F2 \quad \text{is injective}$$



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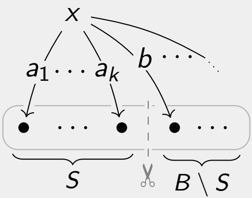

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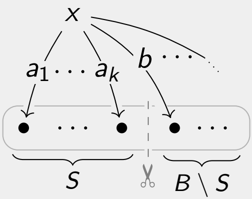

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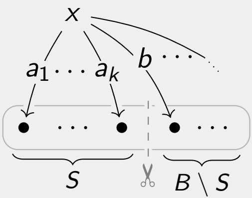

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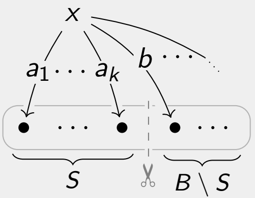

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Theorem: If the functor F is cancellative, then we can replace

$$\ulcorner F\chi_S^B(c(x))\urcorner(\delta, \beta) \quad \text{with} \quad \ulcorner F\chi_S(c(x))\urcorner(\delta) \qquad (F2 \subseteq F3)$$

Conclusions

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Hennessy-Milner-Theorems for free

- for $\top$, $\wedge$, $\neg$, $\ulcorner t \urcorner/2$ ($t \in F3$) if F is zippable.
- for $\top$, $\wedge$, $\ulcorner t \urcorner/1$ ($t \in F2$) if F is zippable and cancellative.

References

[Pat03] Dirk Pattinson. “Coalgebraic modal logic: soundness, completeness and decidability of local consequence”. In: **Theoretical Computer Science** 309.1 (2003), pp. 177–193. DOI: https://doi.org/10.1016/S0304-3975(03)00201-9.

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